Anisotropic Lattices and Dynamical Fermions *

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We report results from full QCD calculations with two flavors of dynamical staggered fermions on anisotropic lattices. The physical anisotropy as determined from spatial and temporal masses, their corresponding dispersion relations, and spatial and temporal Wilson loops is studied as a function of the bare gauge anisotropy and the bare velocity of light appearing in the Dirac operator. The anisotropy dependence of staggered fermion flavor symmetry breaking is also examined. These results will then be applied to the study of 2-flavor QCD thermodynamics.

1. INTRODUCTION

Anisotropic lattices have been used extensively for $T = 0$ calculations[1]. Finite temperature calculations using anisotropic lattices exploit the natural asymmetry of finite temperature field theory to reduce lattice spacing errors associated with the transfer matrix at less cost than is required for the full continuum limit[2].

With a sufficiently small value for the temporal lattice spacing, a_t, we can vary the temperature in small discrete steps by varying the number of time slices, N_t. In this study we sweep through the transition by studying different N_t's (8–64). Varying the temperature at fixed temporal and spatial lattice spacing separates temperature and lattice spacing effects, allowing a study of the temperature dependence with all other parameters fixed.

The coefficients needed to relate lattice observables to the physical energy and pressure are determined as a by-product of the zero temperature studies needed to choose the bare parameters. Once determined, these "Karsch" coefficients[3] can be used for all temperatures since they depend only on the intrinsic lattice parameters and not on N_t. This allows a straight-forward determination of the temperature dependence of the energy and pressure, again at fixed lattice spacing. With two or more slightly different values for a_t, a high-resolution sampling of temperatures can be investigated.

As the temporal lattice spacing, a_t, approaches the continuum limit ($a_t \to 0$), the part of the flavor symmetry, which is violated by terms of $\mathcal{O}(a_t^2)$, is expected to be restored. In this study we are examining our data for evidence of improvement of the flavor symmetry, when a_t becomes sufficiently small.

2. THE ANISOTROPIC STAGGERED ACTION

We are simulating full QCD with two dynamical flavors of staggered fermions on an anisotropic lattice. Our calculations are based on the QCD action $S^\xi = S^\xi_G + S^\xi_F$, where the gauge action is:

$$S^\xi_G = -\frac{\beta}{N_c}\frac{1}{\xi_o}\left[\sum_{x,s>s'} P_{ss'}(x) + \xi_o^2 \sum_{x,s} P_{st}(x)\right],$$

and the fermion action is:

$$\begin{aligned} S^\xi_F &= \sum_x \overline{\psi}(x)\left[m_f + \nu_t \not{D}^{\text{Staggered}}_t\right]\psi(x) \\ &+ \sum_x \overline{\psi}(x)\left[\frac{1}{\xi_0}\sum_s \not{D}^{\text{Staggered}}_s\right]\psi(x). \end{aligned}$$

In our simulations we attempt to examine the QCD phase transition for volumes ($16^3\times 4$), quark masses ($m_\pi/m_\rho \approx 0.3$) and a spatial lattice spacing ($a_s \approx 0.3$ fm) similar to those used in $N_t = 4$, 2-flavor thermodynamic studies on isotropic lattices. Thus, we adjust the bare bare anisotropy

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(ξ_0) and the renormalization of the speed of light (ν_t) to yield the required a_s and m_π/m_ρ but work with a much smaller temporal lattice spacing choosen so the critical value of N_t is approximately 16.

3. SIMULATIONS

run	*volume*	*traj.*	β	ξ_0	m_f
1	16^3x32	5800	5.425	1.5	0.025
2	16^2x24x32	5100	5.425	1.5	0.025
3	16^2x24x64	1300	5.695	2.5	0.025
4	16^2x24x64	1400	5.725	3.44	0.025
5	16^2x24x64	3400	5.6	3.75	0.025
6	16^2x24x64	3200	5.3	3.0	0.008
7	16^3x24	3500	5.3	3.0	0.008
8	16^3x20	2800	5.3	3.0	0.008
9	16^3x16	4500	5.3	3.0	0.008
10	16^3x12	8200	5.3	3.0	0.008
11	16^3x8	5500	5.3	3.0	0.008

Table 1
Parameters of all the calculations. All runs have dynamical $\nu_t = 1.0$ except run 3 which has $\nu_t = 1.2$.

We have used the zero temperature runs 1-6 for scale-setting and determination of the anisotropy. For all the zero-temperature runs the renormalized anisotropy ξ_r is calculated both from the ρ masses in the spatial and temporal directions and from matching the static potentials[4]. Figure 1 shows that both methods provide values for ξ_r which are reasonably close.

The idea behind run 7 through 11 is that we keep the spatial lattice spacing, a_s, and all other run parameters constant and change only the number of lattice points, N_t, in the temporal direction.

Our simulations implement the R-algorithm[5] with step–size $\Delta t = 0.005$ for all jobs from Table 1. Figure 2 shows that our choice of Δt is consistent with the requirements for small finite step–size error on the physical quantities.

4. THE VELOCITY OF LIGHT

The velocity of light on a lattice can be defined through the meson dispersion relation:

$$E^2_{t,phys}(p_s) \quad = \quad \frac{E^2_{t,lat}(0)}{a_t^2} + c^2_{ts}(p_s)P^2_{s,lat}\frac{1}{a_s^2}$$

We tune ν_t, so that $c_{ts}(p_s) \approx 1$. The velocity of light is calculated for the π propagating in the temporal direction and having non–zero momentum for three values of ν_t: 1.0, 0.8 and 1.2, where the first is a dynamical parameter and the last two are valence parameters. From Figure 3 we see that the choice of $\nu_t = 1.0$ gives velocity of light closest to 1.0.

5. IMPROVEMENT OF THE FLAVOR SYMMETRY

Table 2 shows the meson masses for run 3 ($a_s = 0.231(7)$ fm, $a_t = 0.072(4)$ fm) and run 4 ($a_s = 0.243(3)$ fm, $a_t = 0.057(2)$ fm).

We choose $\Delta_\pi = (m_{\pi_2} - m_\pi)/m_\rho$, where π_2 is the second local staggered pion, as a quantitative measure of the flavor symmetry breaking in the spatial (S) and temporal (T) directions. The data shows that in the temporal direction for both runs Δ_π is smaller than its value in the spatial direction, which means that we are seeing improvement of the flavor symmetry as a_t becomes finer. Especially for run 4, the π and π_2 look virtually degenerate.

am	*S, #3*	*T, #3*	*S, #4*	*T, #4*
am_π	0.680(3)	0.214(2)	0.771(4)	0.181(2)
am_{π_2}	0.759(12)	0.219(2)	0.836(21)	0.181(2)
am_ρ	0.902(26)	0.283(4)	0.948(13)	0.222(4)
Δ_π	0.09(1)	0.017(9)	0.07(2)	0.000(2)

Table 2
Meson masses for run 3 and 4.

6. THE PHASE TRANSITION

The thermodynamics runs 7–11 have volume $16^3 \times N_t$, where N_t = 8, 12, 16, 20 and 24. The bare parameters are kept constant for all runs 6–11. Figure 4 shows the sweep through the phase transition as we gradually change the temperature by varying only N_t. We fit our data to a hyperbolic tangent form and determine $T_c \approx 160$ MeV from the inflection point.

7. CONCLUSIONS

We have studied the finite temperature QCD phase transition using staggered fermions on an anisotropic lattice with anisotropy of ≈ 4. This allows us to explore the temperature dependence of the transition with all other parameters fixed. The results are roughly consistent with earlier, isotropic studies, showing a value of the critical temperature of approximately 160 MeV. While this approach naturally reduces finite lattice spacing errors associated with a_t, we plan to include improvements to the spatial parts of the staggered fermion action so that the $\mathcal{O}(a_s^2)$ errors are reduced as well.

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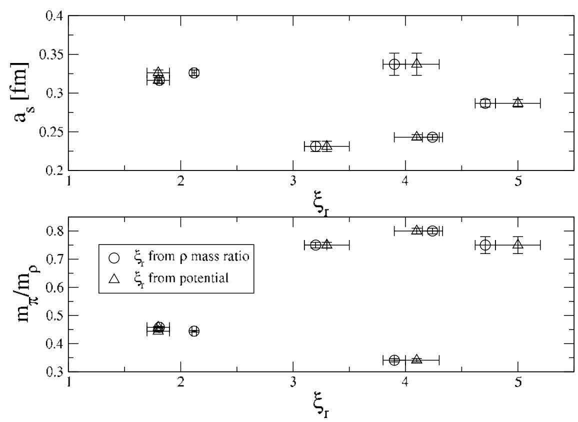

Figure 1. Scatter plots of the zero temperature runs 1–6.

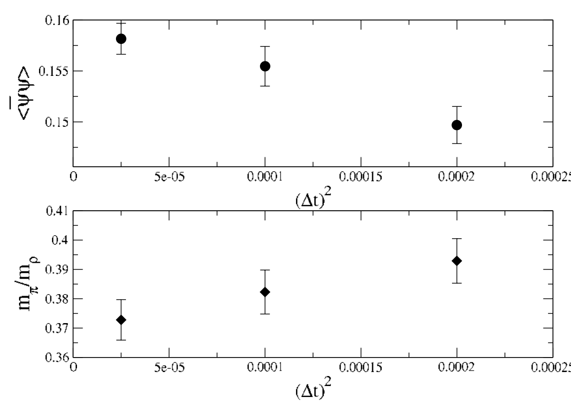

Figure 2. Step–size test. Volume $8^3 \times 32$, $\beta = 5.35$, $\nu_t = 1.0$, $\xi_0 = 3.5$, $m_f = 0.006$.

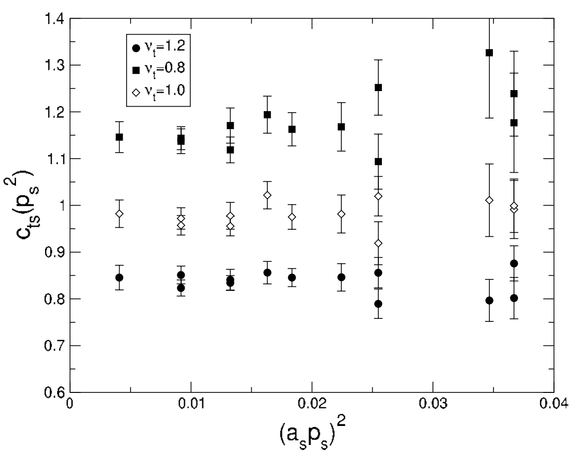

Figure 3. Tuning of the velocity of light using a $16^2 \times 24 \times 64$ volume, $\beta = 5.3$, $\xi_0 = 3.0$, $m_f = 0.008$.

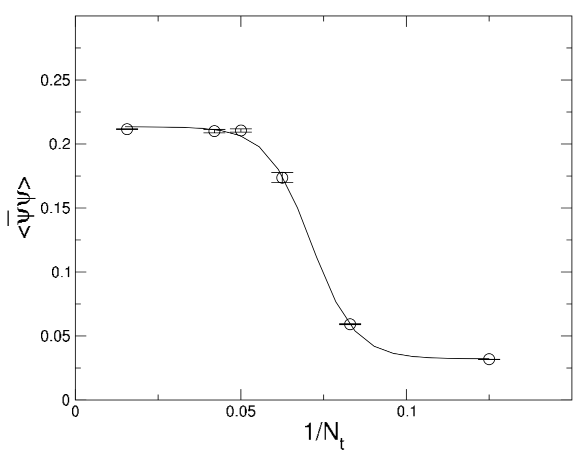

Figure 4. The temperature dependence of $\langle\overline{\psi}\psi\rangle$ in the region of T_c.